Nonlinear mode veering for enhanced resonant sensing

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Abstract: A research trend in micro systems for resonant sensing is a sensitivity enhancement utilizing nonlinear and coupling effects, which is mainly applied to gas sensing and timing applications. In this article, we propose such an enhancement of the sensitivity for an atomic force microscopy probe. This scheme is based on a two beam array, where one beam is active and acting as probe, while the other beam is passive. Both beams of this array are designed to have an identical resonance frequency at a defined distance between the active beam and a surface, in the mechanically uncoupled case. The occurring mode localization close to this distance and the nonlinear interaction potential lead to an increased sensitivity in this region, with respect to frequency change and amplitude ratio. The proposed scheme is experimentally validated with a macro-scale test setup, mimicking a microscale system.

1. Introduction

Today, one challenge in micro-electro-mechanical systems (MEMS) for resonant sensing is to further increase their performance metrics. These performance metrics include increasing the sensitivity to measure changes of quantities and the time required to measure such changes. For instance, atomic force microscopy (AFM) in its dynamic operation modes is based on resonant sensing. Figure 1 a) shows the setup of an AFM, wherein a probe is excited at its resonance frequency and brought in close proximity to a sample surface. A positioning stage is moving the sample in a raster scan pattern to measure the topography of the sample. A change in tip-sample distance Δd_0 leads to a change in the probe's resonance frequency $\Delta\omega_0$, due to nonlinear interaction forces [7], as illustrated in Fig. 1 b). This change in resonance frequency, or the resulting amplitude or phase change, is the measured quantity, and its change with respect to the change in tip-sample distance is the corresponding sensitivity (e.g. $\Delta\omega_0/\Delta d_0$). For instance, in an AFM process the tip-sample distance d_0 is controlled to maintain a set resonance frequency ω_0 by adjusting the positioning stage's vertical $\bar{z}$-axis. Thus, the $\bar{z}$-axis motion of the positioning stage represents the topography of the sample relative to a reference point. Typical amplitudes of the probe and tip-sample distances are in the range of tens to a few hundreds of nanometers and are of the same order of magnitude [7].

Approaches to increase the performance metric of an AFM are mainly based on a linear methodology (e.g. focused on the probe's resonance frequency and damping) and have

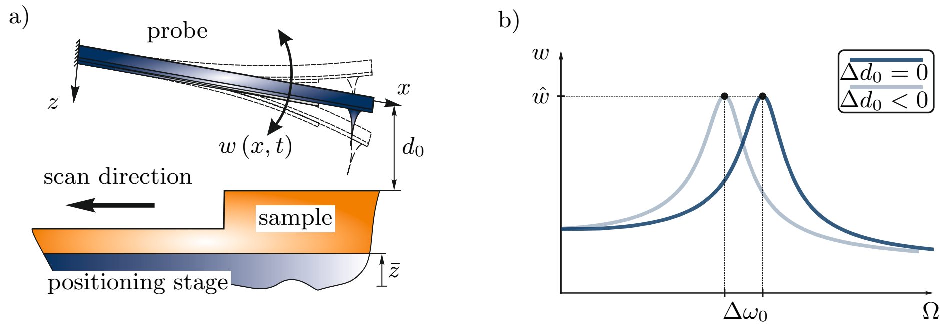

Figure 1. a) Basic AFM setup [13]. b) Shift of resonance frequency $\Delta\omega_0$ due to a change in tip-sample distance Δd_0.

reached their upper best. Recent approaches concentrate on utilizing nonlinear dynamic phenomena to significantly go beyond the performance of what can be achieved by linear methodology. Prakash et al. [11] utilized a parametric resonance, introduced by a control loop, to increase the sensitivity. The parametric resonance has a very high Q-factor and, thus, enhances the amplitude change per change in tip-sample distance $\Delta\hat{w}/\Delta d_0$. Jeong [8] used a special probe design in which an in-lay beam is integrated in a cantilever to realize an internal resonance, which is triggered by a change in tip-sample distance, also enhancing $\Delta\hat{w}/\Delta d_0$. However, approaches based on nonlinear dynamics applied to AFM are limited and require further investigation. Nonetheless, the potential of leveraging nonlinear dynamics in resonant sensing has been shown for MEMS in general. Among others, approaches aim to utilize parametric [12] and internal resonances [2, 13], modal interaction [16], frequency hardening [3, 15] as well as bifurcations [4] to increase sensitivity, reduce phase noise and achieve an increased detection speed.

Apart from the utilization of nonlinear dynamics, coupling effects in arrays of beams are a promising research area to achieve an increased sensitivity. This sensitivity increase is mainly based on mode-localization [14, 17], which is a spatial confinement of energy within a system of coupled oscillators. In this case, a symmetry breaking perturbation (e.g. stiffness/mass change) can be measured with orders of magnitude increased sensitivity. However, utilizing nonlinearities or coupling effects is mainly applied to timing or mass detection applications.

In conclusion, recent research illustrates the potential of utilizing nonlinear dynamics and coupling effect to increase the performance metric of MEMS. However, consideration of those approaches to AFM are rare and a detailed study of these effects is missing.

In the scope of this article, a scheme is presented to use the *nonlinear interaction* between an AFM probe's tip and a surface together with a mechanical coupling effect between two beams to achieve an increased sensitivity ($\Delta\omega_0/\Delta d_0$). The mechanical coupling between both cantilevers is caused by the clamp as their common base, seen in Fig. 2. One beam is active, i.e. is excited at its resonance frequency and the other is passive. The active beam undergoes the nonlinear interaction between tip and sample surface. For experimental validation of this approach, a macro-scale test setup has been build to emulating a micro scale AFM [5]. This setup allows for specific adjustments of the array's parameters (e.g. coupling, resonant frequencies, tip-sample distance). The following section begins with the model used for the analysis, followed by the macro-scale test setup, the analysis and experimental results. Finally, conclusions from the results and future work is presented.

2. Model

The considered system is an array of two cantilever beams, connected with a common base. Each beam has an integrated actuator and sensor and is subjected to a quadratic interaction force between probe tip and sample. Such interaction forces may be due to e.g. van der Waals or permanent magnet forces. Note that this type of sensor and actuator is especially for array technology in AFM, and differs from the commercially available AFM probes [10]. As illustrated for a single beam in Fig. 1, the displacement of the beam towards the sample surface is $w\,(x,t)$, which is also the coordinate the equations of motion are derived for. An extended Hamilton's principle is used to derive the partial differential equations (PDEs) describing the motion of the coupled two beam array, under Bernoulli assumptions. For the discretization of the system of PDEs, a Galerkin method (with $w_m\,(x,t) = \sum_n W_n\,(x)\,X_n\,(t)$) has been applied. The displacement of beam m is given by w_m, where W_m and X_m are the spatial mode shapes (i.e. comparison functions) and the time varying modal displacements of the m-th beam, respectively. A detailed derivation can be found in [5, 6]. Thus, the reduced order equations of motion including first intrinsic bending modes of both beams are given in matrix form by

$$\begin{aligned}&\begin{bmatrix}\ddot{X}_1\\ \ddot{X}_2\end{bmatrix}+\begin{bmatrix}W_{\mathrm{c}11} & W_{\mathrm{c}12}\\ W_{\mathrm{c}21} & W_{\mathrm{c}22}\end{bmatrix}\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}+\begin{bmatrix}W_{\mathrm{k}11} & W_{\mathrm{k}12}\\ W_{\mathrm{k}21} & W_{\mathrm{k}22}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}-\begin{bmatrix}W_{\mathrm{F}11} & W_{\mathrm{F}12}\\ W_{\mathrm{F}21} & W_{\mathrm{F}22}\end{bmatrix}\begin{bmatrix}AC_1F_{\mathrm{ext}}\\ AC_2F_{\mathrm{ext}}\end{bmatrix}\\ &\quad-\begin{bmatrix}W_{\mathrm{NL}11} & W_{\mathrm{NL}12}\\ W_{\mathrm{NL}21} & W_{\mathrm{NL}22}\end{bmatrix}\begin{bmatrix}\dfrac{\tau_m}{(\hat{d}_{01}-W_1X_1)^2}\\ \dfrac{\tau_m}{(\hat{d}_{02}-W_2X_2)^2}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix},\end{aligned} \tag{1}$$

where $W_{\mathrm{c}mn}$ are the coupled damping terms, $W_{\mathrm{k}mn}$ are the coupled stiffness terms, $W_{\mathrm{F}mn}$ are the coefficients of the actuation term applied to each beam, $W_{\mathrm{NL}mn}$ are the coefficients of the nonlinear tip-sample interaction forces applied to each beam. These coefficients include the integrals of the comparison functions used for the Galerkin approach [5,6]. τ_m is the coefficient of the attractive force potential between tip and sample, and d_{0m} is the separation distance between the undeflected cantilever m and the sample surface (see Fig. 1). The forcing function F_{ext} is the nondimensional oscillatory component of the excitation signal, with the dimensional amplitude applied to beams 1 and 2 defined by AC_1 and AC_2, respectively. For frequency modulation AFM (FM-AFM), F_{ext} needs to be defined such that the excitation signal is sinusoidal with a phase lead of 90° over the measured output signal, which in this case will be the tip displacement $w_1\,(l,t)$ [1]. The resulting periodic response of the system as a function of tip-sample separation d_0 can be found by solving Eq. (1) in the time domain. The parameters used for the numerical analysis of Eq. (1) in the following sections can be found in [5].

3. Experimental setup

To simulate the operation of an AFM array, an equivalent macro-scale experiment is utilized. The experiment is scaled up 1000× in comparison to standard AFM cantilever dimensions (hundreds of micrometers in length). Using a macro-scale test rig allows key parameters of the system, including coupling strength and individual cantilever dimensions, to be easily varied for the purpose of studying their influence on system response. The macro-scale test rig is depicted in Fig. 2, and consists of a variable number of cantilevers clamped to a base structure. Each cantilever is equipped with a piezo-film actuator and strain gauge sensors. Permanent magnets at the tip of each cantilever are used to simulate attractive tip-sample interaction forces. The tip-sample separation distance is altered using a stepper motor assembly.

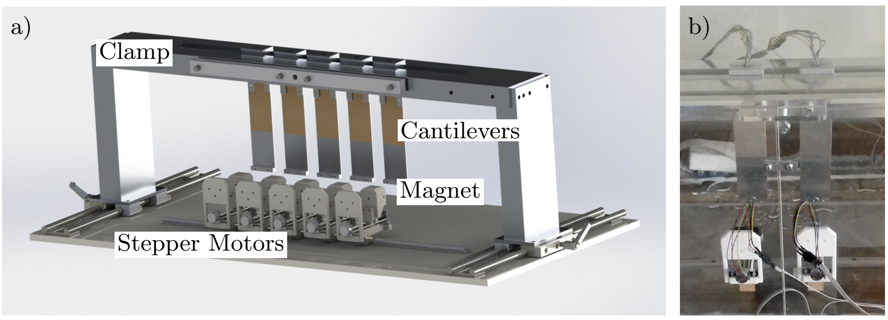

Figure 2. a) Rendering of the macro-scale test rig. b) Detail view of the real test rig.

To actuate the system at resonance for FM-AFM, a PID controller is employed to control a 90° phase angle between displacement the w_1 of beam 1 (measured by strain gauges) and the excitation signal F_{ext}. The amplitude of the actuation signal is user controlled and constant in this case. The stepper motors are used to move permanent magnets relative to the cantilevers. This allows the tip-sample separation for each cantilever in the array to be altered independently. As with standard FM-AFM, the actuation frequency changes as a function of tip-sample separation, allowing actuation frequency to be used as a measurement variable. The macro-scale experiment was used as a proof of concept of the proposed method of sensitivity enhancement utilizing a two beam array instead of standard single cantilever techniques.

4. Results

The active beam is excited with an amplitude AC_1 at its resonance with FM control and is acting as a probe in close proximity to a surface. The passive beam is neither brought in close proximity nor excited. The effects utilized for the sensitivity increase are based on the nonlinear interaction potential between beam 1 and the surface, as well as the mode localization between the active and the passive beam. Thus, the nonlinear shift to lower resonance frequencies of beam 1 is used to trigger and break the symmetry of the array to achieve an increased frequency change per change in tip-sample distance. This general

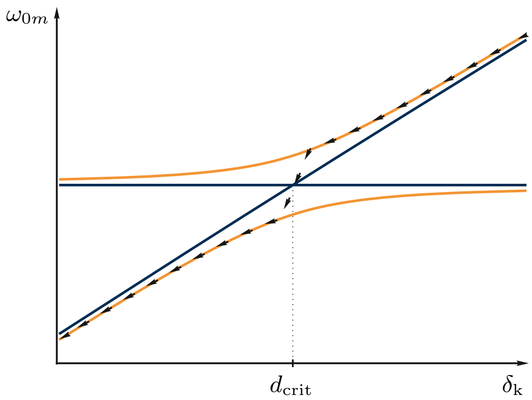

Figure 3. Mode veering for a two beam array, with eigenfrequencies ω_m, the stiffness detuning δ_{k} and the critical distance d_{crit}. Blue lines depict mechanically uncoupled modes (ω_{0m}), orange lines are out-of-phase (upper) and in-phase (lower) array mode, black arrows indicate path taken in FM-AFM.

concept is illustrated in Fig. 3, showing the eigenfrequencies ω_m over a stiffness detuning δ_{k} at beam 1. In the mechanically uncoupled case (blue lines) both frequencies (ω_{01} and ω_{02}) are identical at d_{crit}, at which $\delta_{\mathrm{k}} = 0$. Changing δ_k leads to a change of ω_{01}, while ω_{02} remains constant. In the coupled case (orange lines) the in-phase (lower frequencies) and out-of-phase (higher frequencies) mode have no intersection, but diverge from another with a varying δ_{k}. This effect is also known as eigenvalue veering [9], and is used in mode-localization sensing schemes. Thus, by controlling the phase between beam 1 and the excitation while decreasing δ_{k} from positive to negative leads to a sudden shift from the out-of-phase to the in-phase mode (arrows in Fig. 3), showing a steeper gradient $\Delta\omega_{\mathrm{a1}}\ \delta_k$ in the neighborhood of d_{crit}.

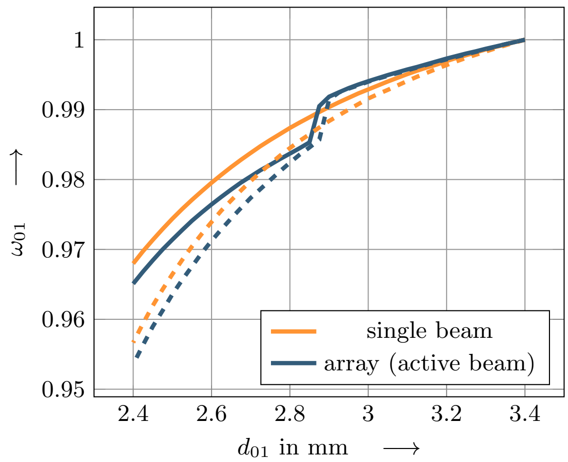

Figure 4. Change of normalized eigenfrequency of a two beam array while approaching a sample surface, damping ratio $\zeta = 0.007$. The amplitude of beam 1 is in the range of 0.5 to 1 mm. Line style represents the excitation amplitude; solid and dashed being $AC_1 = 0.001$ and $AC_1 = 0.002$, respectively.

Figure 4 shows a numerical simulation (model with FM-AFM) of the approach of the active beam to a fixed surface; depicted is its change of eigenfrequency (scaled) ω_{01} over the tip-sample distance d_{01}. The sharp change of eigenfrequency occurs at a critical distance $d_{\mathrm{crit}} \approx 2.875$ mm, which is not present in the approach curve of a single beam (orange line in Fig. 4). At d_{crit} the eigenfrequencies of the uncoupled beams are closest together. Every symmetry breaking detuning of parameters (e.g. stiffness of beam one in this case), leads to a divergence of the coupled eigenvalues of the array modes (i.e. in-phase and out-of-phase, c.f. Fig. 3). This mode veering, together with the nonlinear potential and the FM controller, leads to an increased sensitivity at this region about d_{crit}. As described earlier, the controller ensures a 90° phase angle between the excitation signal and w_1, leading to a change between out-of-phase and in-phase mode of the array, when crossing d_{crit}.

The conditions to define d_{crit} are determined by the uncoupled resonance frequencies of the two beams as well as the excitation amplitude AC_1. The influence of an increased

AC_1 can be observed as a shift of d_{crit} to higher values (Fig. 4). This shift is due to the amplitude dependent resonance frequency of the active probe (e.g. caused by the nonlinear potential). Thus, AC_1 influences the slope of the whole approach curve, also in case of the single beam. Additionally, the damping ratio ζ as well as the coupling between the beams influence the sensitivity. A decrease of ζ increases $\Delta\omega_{01}/\Delta d_{01}$ at d_{crit}, whereas a weaker coupling decreases it. A stronger coupling leads to an increased slope, until multiple coexisting solutions appear, connected by two saddle node bifurcations [5]. This behavior has also been shown experimentally. Figure 5 shows the frequency change of ω_{01} while approaching a permanent magnet with the array concept and a single beam with the macro-scale test setup. As can be seen, the characteristics reassemble those gained from numerical simulations (c.f. Fig. 4), resulting in an increased sensitivity in a region close to d_{crit} for the array configuration.

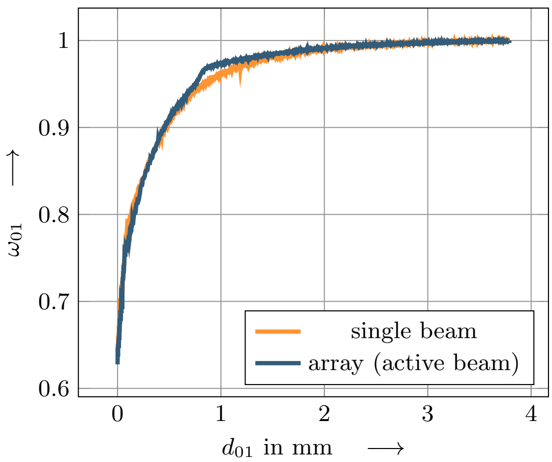

Figure 5. Experimental approach curves.

Another measure to increase the probe's sensitivity is the amplitude ratio between the two beams. As shown in Fig. 6, considering the system below d_{crit} (at which the amplitude ratio is equal to 1) leads to a strong increase of this ratio. In contrast to schemes used with classical mode localization sensing, the increase in this case is nonlinear, due to the interaction potential between tip and sample surface. The slope of this increase also depends on the excitation amplitude AC_1 as well as on the stiffness of the coupling between the beams. In this case, higher amplitudes result in an increased sensitivity. In contrast to the frequency slope shown in Fig. 4, the slope of the amplitude ratio increases in case of a decreased coupling. Thus, very low coupling results in a very high sensitivity, as also used in other schemes of localized mode sensing [17]. This strong increase for weakly coupled beams is due to a strong change in eigenvectors of the array modes due to mode veering [9].

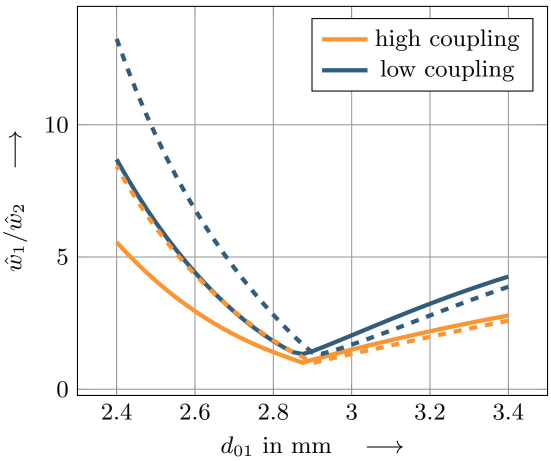

Figure 6. Amplitude ratio between the two beam's of an array while approaching a sample surface, damping ratio $\zeta = 0.007$. Line style represents the excitation amplitude; solid and dashed being $AC_1 = 0.001$ and $AC_1 = 0.002$, respectively.

5. Conclusion

The approach presented in this article to increase the sensitivity of an AFM probe is based on utilizing nonlinear and coupling effects with a two beam array. This array consists of an active beam sensing the actual topography changes and a passive beam for sensitivity enhancement. The occurring dynamic effect can be levered in two ways, first by using the increased frequency change over the tip-sample distance $\Delta\omega_{01}/\Delta d_{01}$ and, second, by utilizing the increased change of amplitude ratio between the two beams of an array $\hat{w}_1/\hat{w}_2$. Both effects show a higher increase in sensitivity for an increased amplitude. However, the frequency change increases for an increased coupling between the beams of the array, whereas the slope of the amplitude ratio decreases in this case. The sensitivity gain of both approaches can be increased by decreasing the damping coefficient ζ as well as by increasing the amplitude of excitation AC_1.

Using an increased $\Delta\omega_{01}/\Delta d_{01}$ has multiple benefits. When controlling the frequency in an AFM operation, the time to detect changes mainly depends on the control scheme used and is, thus, not limited by the decay coefficient of the individual beams. Additionally, the increased frequency change can be further magnified by using an amplitude detection scheme, or utilizing parametric resonance [11,12], which would combine an increased frequency change with an increased amplitude change.

On the other hand, in combination with a very weak coupling, the amplitude ratio can be used to gain an orders of magnitude increased sensitivity compared to the frequency change. In this case, the probe is operated below the critical distance d_{crit} at a high slope. In contrast to most sensing schemes using mode localization, the probe in close proximity is driven at resonance using a phase locked loop (PLL) and both amplitudes are measured. Thus, there is no need for an observer base mode estimation, resulting in a faster control scheme. The influence of the nonlinear change of the amplitude on the image quality in an AFM process can be neglected, if the positioning stage is used in the PLL to maintain a phase shift of 90° between excitation and beam response.

Future work is devoted to an analytical solution for the dependency of d_{crit} on the systems's parameters (e.g. AC_1, ζ, coupling strength), the achievable sensitivity as well as the forces exerted onto a sample. Moreover, the implications gained are transferred to AFM array, in which all beams are actively probing the sample surface. In this case, higher modes of the array are considered in order to explain observed coupling phenomena while imaging a sample.

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